TARGET MATHEMATICS

THE EXCELLENCE KEY

AGYAT GUPTA (M.Sc., M.Phil.)

21st YEAR OF QUALITY EDUCATION

CODE:- AG-TS-11-0036 REGNO:-TMC -D/79/89/36

GENERAL INSTRUCTIONS :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
5. Use of calculator is not permitted.
6. **Please check that this question paper contains 5 printed pages.**
7. **Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.**

सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 5 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

Pre-Board Examination 2012 -13

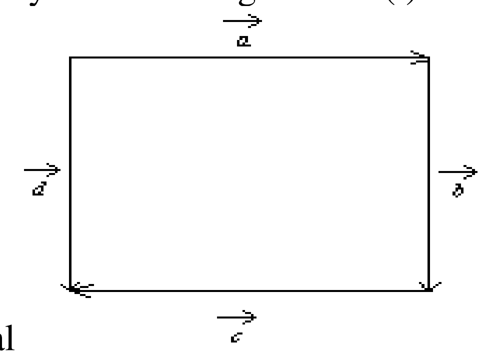
Time : 3 Hours	अधिकतम समय : 3
Maximum Marks : 100	अधिकतम अंक : 100
Total No. Of Pages :6	कुल पृष्ठों की संख्या : 6

CLASS – XII CBSE MATHEMATICS

PART – A

Q.1 Find the value of $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$. Ans $=\frac{-\pi}{3}$

Q.2 In figure (a square), identify the following vectors.(i) Coinitial (ii) Equal



(iii)Collinear but not equal Ans $.(i)a\ \&\ d,(ii)b\ \&\ d,(iii)a\ \&\ c$

Q.3 Find the slope of the tangent to the curve $x=t^2+3t-8, y=2t^2-2t-5$ at the point.(2, -1) Ans $=\frac{6}{7}$

Q.4 If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. Ans $\lambda=8$

Q.5 If f : R→ R be defined as f(x) = $\frac{3x+7}{9}$, then find $f^{-1}(x)$. Ans $f^{-1}(x)=\frac{9x-7}{3}$.

Q.6 Let relation $R=\{(x,y)\in w\times w: y=2x-4\}$. If (a , - 2) and

	$(4, b^2)$ belong to relation R, find the value of a and b . Ans. a=1,b=2
Q.7	Find values of k if area of triangle is 4 square units and vertices are (k,0),(4,0),(0,2). Ans k=0,8
Q.8	The number of all possible matrices of order 3×3 with each entry 0 or 1 . Ans $=2^9$
Q.9	Write the total number of binary operation on a set consisting of n element . ANS n^{n^2}
Q.10	If the points (1, 1, p) and (-3, 0, 1) be equidistant from the plane $\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of p. Ans $p=1,\frac{7}{3}$
	PART – B
Q.11	Show that the curve $y^2 = 8x$ & $2x^2 + y^2 = 10$ intersect orthogonally at the point $(1, 2\sqrt{2})$. Ans $m_1 \times m_2 = -1$
Q.12	If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a ΔABC respectively. Find an expression for the area of ΔABC and hence deduce the condition for the points A, B, C to be collinear. $area of \Delta ABC = \frac{1}{2}\left|\vec{AB}\times\vec{BC}\right| \Rightarrow A(\Delta ABC)=0 \therefore \vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}=0$
Q.13	Evaluate: $\int e^x Sin^2 4x dx$. Ans $\frac{e^x}{2}-\frac{e^x\cos 8x}{130}-\frac{4e^x \sin 8x}{65}$ OR

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	Evaluate : $\int e^x \left(\frac{x^2+1}{(x+1)^2}\right)dx$. Ans $e^x - \frac{2e^x}{x+1}$
Q.14	Find all point of discontinuity of f , where f is defined as following : $f(x)=\begin{cases}\lvert x\rvert+3 & if x \le -3\\ -2x & -3<x<3\\ 6x+2 & if x \ge 3\end{cases}$. **Ans** $f(x)=\begin{cases}-x+3 & x\le -3\\ -2x & -3<x<3\\ 6x+2 & x\ge 3\end{cases}$ **f(x) is continous at x = - 3 Whe ; RHL=LHL = FUNCTIONAL VALUE = 6 & f(x) is not continous at x = 3 ; RHL = 20 & LHL = - 6**
Q.15	Show that the following differential equation is homogeneous, and then solve it : $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$. Ans $\frac{\left(\log\frac{y}{x}-1\right)^2}{\left(y\log\frac{y}{x}-\frac{y}{x}\right)} = xc$
Q.16	The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds. Ans $r=(63t+27)^{\frac{1}{3}}$ **OR** Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x (x\neq 0)$ given that y = 0 when $x=\frac{\pi}{2}$. Ans $y \sin x = x^2 \sin x - \frac{\pi}{4}$
Q.17	Prove the following : $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$.

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Q.18 Prove that: $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.

Q.19 Suppose 15% of men and 36% of women have grey hair. The probability of dying hair by men is 21% and by women is 63%. A dyed hair person is selected at random, what is the probability that this person is a women? Excessive use of dyes to colour the hair can prove harmful. Elaborate.

Ans: dyes may contain harmful inorganic compounds which can degrade the quality of hair and although they provide style but might resul 3 hair problems like hair fall. Split end etc.

OR

A survey revealed that 70% men and 30% women eat pan-masala. 10% of these men and 20% of these women eat brand X pan-masala. What is the probability that a person seen eating brand X will be a man? Why would you discourage intake of pan-masala? Ans :

$$P(M \mid E) = \frac{P(M)P(E \mid M)}{P(M)P(E \mid M) + P(W)P(E \mid W)},$$

where E: event of taking brand X.

$$\Rightarrow \quad = \frac{(70/100).(10/100)}{(70/100).(10/100) + (30/100).(20/100)}.$$

Reason: Intake of pan-masala could be highly injurious to health. It causes cancer. So we would discourage its intake.

Q.20 Discuss the relation R in the set of real number , defined as $R = \{(a,b): a \le b^3\}$ is Reflexive , Symmetric & Transitive . Ans ; Not reflexive $\frac{1}{2} \rangle \frac{1}{8} \Rightarrow \frac{1}{2}, \frac{1}{8} \in R \therefore \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$; symmetric $(1,3) \in R \Rightarrow (1,3) \notin R$ & not transitive $(100,5) \in R \,\&\, (5,2) \in R \Rightarrow (100,2) \notin R$

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Q.21 If $y = \frac{x \sin^{-1} x}{\sqrt{(1-x^2)}} + \log\sqrt{1-x^2}$. Prove that $\frac{dy}{dx} = \frac{\sin^{-1} x}{(1-x^2)^{3/2}}$.

OR

Prove that the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at x = 0, is ¼ .

4

Q.22 Vectors $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are non – coplanar. The position vectors of the points A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{j} + 2\hat{k}$ respectively. Find the position vectors of a point P on the line AB and a point Q on the line CD such that $\vec{PQ}$ is perpendicular to both $\vec{AB}$ and $\vec{CD}$. ANS. $P = 3\hat{i} + 8\hat{j} + 3\hat{k}$ and $Q = -3\hat{i} - 7\hat{j} + 6\hat{k}$.

PART – C

Q.23 If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, *find* $(AB)^{-1}$ Ans

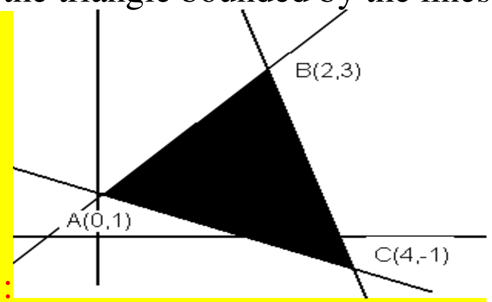
$$(AB)^{-1} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}, B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Q.24 A toy manufacturers produce two types of dolls ; a basic version doll A and deluxe version doll B. Each doll of type B takes twice as long to produce as one doll of type A . The company have time to make a maximum of 2000 , dolls of type A per day , the supply of plastic is sufficient to produce 1500 dolls per day and each type requires equal amount of it .The deluxe version i.e. type B requires a fancy dress of which there are only 600 per day available . If the company makes profit of ₹ 3 and ₹ 5 per doll respectively on doll A and B , how many of each should be produced weekly in order to maximize the profit ? Solve it by graphical method. Ans : z = 3x + 5 y

$x+2y\le 2000, x+y\le 1500, y\le 600; x, y\ge 0$. corner points : (0,0) ; (1500,0) (1000, 500) (800 , 600) & (0 , 600) Thus Z is maxmium at (1000 , 500) and maximum value is 5500 .

Q.25 Evaluate: $\int_0^{\pi} \frac{x}{a^2 - \cos^2 x} dx.$ Ans. $\frac{\pi^2}{2a\sqrt{a^2-1}}$

Q.26 Using integration, find the area of the triangle bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7. Ans :



5

$A_1 = \int_{-1}^{3} \frac{7-y}{2} dy; A_2 = \int_{1}^{3} (1+y) dy; A_3 = \int_{-1}^{1} (2-2y) dy \Rightarrow A_1 - A_2 - A_3 = 6 unit^2$

Q.27 A die is thrown three times. Events A and B are defined as below:
A : 4 on the third throw
B : 6 on the first and 5 on the second throw
Find the probability of A given that B has already occurred. Ans :

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Solution The sample space has 216 outcomes.

Now $A = \begin{Bmatrix} (1,1,4) & (1,2,4) & \ldots (1,6,4) & (2,1,4) & (2,2,4) & \ldots & (2,6,4) \\ (3,1,4) & (3,2,4) & \ldots (3,6,4) & (4,1,4) & (4,2,4) & \ldots & (4,6,4) \\ (5,1,4) & (5,2,4) & \ldots (5,6,4) & (6,1,4) & (6,2,4) & \ldots & (6,6,4) \end{Bmatrix}$

$B = \{(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)\}$

and $A \cap B = \{(6,5,4)\}$.

Now $P(B) = \frac{6}{216}$ and $P(A \cap B) = \frac{1}{216}$

Then $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$

OR

6

In an examination, 10 questions of true- false type are asked. A student tosses a fair coin and determine his answer to each question. If the coin falls heads, he answers true and if it falls tails, he answers false. Show that the probability that he answers at most 7 questions correctly is $\frac{121}{128}$.

ANS : P = 1 / 2 ; q = 1 / 2 .& n = 10 .required probability = at most 7 question correctly it =

$p(x\le 7) = 1 - p(x\ge 8) = 1 - [p(=8) + p(x=9) + p(x=10)]$

$1 - \left[10c_8\left(\frac{1}{2}\right)^{10} + 10c_9\left(\frac{1}{2}\right)^{10} + 10c_{10}\left(\frac{1}{2}\right)^{10}\right] = \frac{121}{128}$

Q.28 State when the line $\vec{r} = \vec{a} + \lambda \vec{b}$ is a parallel to the plane $\vec{r}.\vec{n} = d$.

Show that the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} + \hat{j} + 4\hat{k})$ is parallel to the plane

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	$\vec{r}\cdot(-2\hat{i}+\hat{k}) = 5$ .Also find the distance between the line and the plane. Explain the importance of a wards for players. Ans: Awards have very high importance for the players. By winning awards, player fell themselves motivated and develop a desire to win again. Ans Required Condition for line // to plane is $\vec{b}\bullet\vec{n}=0$ and distance between plane and line $\frac{7}{\sqrt{5}}$
Q.29	Find the shortest distance of the point $(0,c)$ from the parabola $y = x^2$, where $0 \le c \le 5$. Ans $S.D. = \frac{1}{2}\sqrt{4c-1}$ OR Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone. Ans $H = h - x\cot\alpha$ $CSA = f(x) = 2\pi RH = 2\pi x(h - x\cot\alpha)$
	***********//**********
	WINNER LOSE MUCH MORE OFTEN THAN LOSERS. SO IF YOU KEEP LOSING BUT YOU'RE STILL TRYING, YOU'RE RIGHT ON TRACK.

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